Planet sizes in UNIVERSE™

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May 5, 2003

1 Introduction

In the role-playing game UNIVERSE™, planets that players might walk upon—terrestrial planets, in other words—range in size from size 0 (a small moonlet or asteroid) to size 9 (a *large* terrestrial planet with extreme gravity). These "sizes" can be thought of as size classes, each connoting a certain area and gravity.

But how do these translate into real-world terms? How big *is* a size 8 world? We can deduce much if we examine the UNIVERSE™ rules closely.

2 Number of environs

First we have the planet maps. Each map shows the "environs" of a world of a given size. The pattern of the planet maps is obvious upon inspection: starting with size 0, we first add one environ to each hemisphere, then a ring of four, then a ring of eight, and so on. Table 1 shows the number of environs for planets from sizes 0 to 9.

Size	Environs
0	1
1	2
2	6
3	10
4	18
5	26
6	38
7	50
8	66
9	82

Table 1: Planet size versus number of environs

A mathematical formula for this is given in Appendix A.

3 Size of an environ

Let us assume for the moment that all environs have the same area. In general, area is proportional to the square of dimension. That is, a square twice as big as another has four times as much area. In particular, the area of a sphere is proportional to the square of its radius, and conversely the radius is proportional to the square root of the area. If each environ has the same area, then the number of environs is proportional to the area, and so the square root of the number of environs should be proportional to the size (radius). The result of these calculations is shown in Table 2.

Size	N	$\sqrt{N}$	$\sqrt{N}/\mathrm{size}$
0	1	1.000	undef
1	2	1.414	1.414
2	6	2.450	1.225
3	10	3.162	1.054
4	18	4.243	1.061
5	26	5.099	1.020
6	38	6.164	1.027
7	50	7.071	1.010
8	66	8.124	1.016
9	82	9.055	1.006

Table 2: Square root of number of environs as a function of size

We can see that the correlation holds fairly well—that is, if we divide the square root of the number of environs by the size, we get a relatively constant value. (It's a little off for size 0 and 1 planets, so we may have to be careful about them later.) So our assumption that each environ is roughly the same area seems correct. But how big is each environ?

There are two ways to calculate the size of an individual environ: we can "work forwards", starting with the diameter of a real planet whose size in UNIVERSE™ terms is already known; or we can "work backwards", examining the rules for information and see if it fits what we've calculated above.

3.1 Working forwards

One planet whose size is known to us is Earth. Since a size 5 world has a surface gravity of 1.0 g, we can assume that Earth is a size 5 world (or very close). We know the diameter of Earth, from which we can calculate the surface area (see Appendix B). Dividing the area by the number of environs (26) gives about 19,661,900 km^2, equivalent to a square about 4434.18 km on a side.

3.2 Working backwards

In the UNIVERSE™ rules, the Detailed Environ Map is a hex field 39 rows by 35 columns, with the distance from one hex to an adjacent hex being 100 km. That would make the map roughly 3900 $\times$ 3500 km, or 13,650,000 km^2. Elsewhere in the rules it is stated that an environ is about 4000 by 4000 km,[1] which would give an area of 16,000,000 km^2.

Using the former figure, a size 5 world would have an area of 354,900,000 km^2, which would make its diameter 10,628.65 km. Using the latter figure, we have an area of 416,000,000 km^2 and a diameter of 11,507.25 km. By comparison, Earth's diameter is 12,756.3 km.

3.3 Meeting in the middle

These calculations produce results that are relatively close to each other. As a compromise, we can choose the middle value and let an environ be 16,000,000 km^2 in area. The formula for calculating planetary diameter based on the number of environs can be derived (see Appendix C).

4 Size and diameter

Using (5) to convert environ count to diameters gives us Table 3.

Size	Diameter (km)
0	2257
1	3192
2	5528
3	7136
4	9575
5	11510
6	13910
7	15960
8	18330
9	20440

Table 3: Size and diameter

5 Known terrestrial planets

Applying (6) to some of the terrestrial bodies in the Solar System, and converting from approximate number of environs to size, gives us Table 4. The "+" or "−" after the size indicates that the body's size is larger or smaller than the UNIVERSE™ size.

This looks to be fairly reasonable: Earth is a slightly large size 5 world, as is Venus; Luna (Earth's Moon) is a slightly large size 1, giving it a surface gravity of

[1] At the time of this writing, I was unable to find the exact rule that gives this figure.

Planet	Diameter (km)	N	Size
Mercury	4,880.	4.676	$2-$
Venus	12,103.6	28.7646	$5+$
Earth	12,756.3	31.9506	$5+$
Luna	3,476.	2.372	$1+$
Mars	6,794.	9.063	$3-$
Europa	3,138.	1.933	$1-$
Titan	5,150.	5.208	$2-$

Table 4: Sizes of the planets in UNIVERSE™ terms

"trace". Since a size 2 world has 0.2 g surface gravity, a size 1 might be expected to have around 0.1–0.15 g, so Luna's 0.16 g is in line with its being a large size 1.

Additionally, Mars would be a small size 3. A size 3 world has 0.4 g surface gravity, and Mars has 0.38 g, so again this works out.

6 Gas giants

It's unlikely that anyone would ever land on a gas giant in UNIVERSE™, but it's possible there might be something of interest in the atmosphere, which would mean you *might* want to map it. But how many environs does it take to map, say, Jupiter?

Based on equation (6) in Appendix C, we can calculate the number of environs needed for each of the Solar gas giants. Table 5 gives the results of those calculations (rounded off to the nearest environ).

Planet	N
Jupiter	4014
Saturn	2853
Uranus	513
Neptune	482

Table 5: Number of environs to map known gas giants

Using equation (1) to calculate size from number of environs, we find that Neptune is a close size 22, Uranus is between 22 and 23, Saturn is between 53 and 54, and Jupiter is a whopping size 85 (or close to it)! Mapping these would be difficult at best: Neptune, the smallest of the four, would have a map with 11 rings of environs on the northern part of the map and 10 on the southern. If each ring is 1 cm wide, that makes each half of the map over 20 cm wide (nearly 8").

Add to this the fact that even "small" features on a gas giant are big. The Great Red Spot, an oval 12,000 $\times$ 20,000 km, would by itself occupy 21 environ maps in a three-by-seven grid!

A Mathematically calculating the number of environs

We can generalize the relation between planet size and number of environs into a mathematical formula. Let s be the size of the planet, and N the number of environs. For $s \geq 2$, each increment of s means adding another ring of environs. Each ring has a number of environs equal to the "ring number" times 4 (the first ring outside the central circular environ is ring number 1). We add equal-sized rings first to the northern hemisphere, then to the southern.

So going from size 1 to size 2, we add a ring of four environs; to go from there to size 3, we add another ring of four. We add a ring of eight to go from size 3 to size 4. In general, then, going from size s to size $s+1$ means adding a ring of $4\left\lfloor \frac{s}{2} \right\rfloor$ environs. (The "brackets" are the "floor" function, which means we divide s by 2 and throw out the remainder.) We add up the number of environs in all the rings, plus two for the northern and southern central environs, and we get the equation in (1).

$$N = 4\left(\sum_{i=2}^{s} \left\lfloor \frac{i}{2} \right\rfloor\right) + 2 \quad (1)$$

(Remember, this holds for $s > 2$.)

B Working forwards: details

The size of the real Earth is about 12,756.3 km.[2] The area A of a sphere of diameter d is

$$A = \pi d^2 \quad (2)$$

so the area of the Earth is about 511,210,000 km^2. In UNIVERSE™, a size 5 world (like Earth) has 26 environs, so by division we find that an environ is about 19,661,900 km^2, equivalent to a square about 4434.18 km on a side.

C Meeting in the middle: details

Let an environ be 16,000,000 (1.6×10^7) km^2 in area. The formula for the area A of a sphere of diameter D is

$$A = \pi d^2 \quad (3)$$

So conversely,

$$d = \sqrt{\frac{A}{\pi}} \quad (4)$$

So the formula for calculating planetary diameter d based on the number of environs N is therefore

[2]From "The Nine Planets: A Multimedia Tour of the Solar System" at `http://seds.lpl.arizona.edu/nineplanets/nineplanets/`.

$$d = \sqrt{\frac{(1.6 \times 10^7)(N)}{\pi}} \tag{5}$$

Conversely, we can calculate N from d:

$$N = \frac{\pi d^2}{1.6 \times 10^7} \tag{6}$$